Extended Abstract

(ω, c)-PERIODICITY AND ITS GENERALIZATIONS ON TIME SCALES: THEORY AND APPLICATIONS TO DETERMINISTIC AND STOCHASTIC DYNAMICS

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Abstract

The study of differential and difference equations is fundamental to understanding the qualitative behavior of dynamical systems arising in science, engineering, biology, and economics. Although many results for differential equations extend naturally to difference equations, others differ significantly from their continuous counterparts. The theory of dynamic equations on time scales highlights these similarities and differences and provides a unified framework that eliminates the need to establish parallel results separately for continuous and discrete systems. In this approach, the domain is a time scale, defined as an arbitrary nonempty closed subset of the set of real numbers. This formulation enables the simultaneous treatment of continuous-time, discrete-time, and hybrid dynamical systems. Motivated by the need to model real-world phenomena that evolve continuously, discretely, or through a combination of both, Hilger introduced the theory of time scales in 1988, thereby unifying continuous and discrete analysis and facilitating the study of dynamical systems on arbitrary time domains.

Within the framework of time scales, the qualitative analysis of dynamic equations, such as periodicity, stability, and asymptotic behavior of solutions, has received significant attention. Periodic behavior is particularly important for modeling recurrent phenomena in population dynamics, neural networks, epidemiology, and control systems. However, classical periodicity is often too restrictive to describe more general repetitive behaviors arising from scaling effects or weighted repetitions. To address these limitations, several generalized notions of periodicity have been introduced. Among them, the concept of (ω, c)-periodicity has emerged as a natural extension of classical periodicity, characterizing functions whose values repeat up to a constant multiplicative factor after a fixed time shift. This generalized periodic structure allows for the modeling of phenomena where the system's amplitude evolves over time while preserving a repetitive pattern, making it particularly suitable for the analysis of nonlinear evolution equations .

An important aspect of (ω, c)-periodicity is that it is not solely a property of the function, but also depends fundamentally on the underlying time domain. Consequently, the structure of the time scale plays a decisive role in determining whether such oscillatory behavior can occur. This observation highlights the necessity of studying (ω, c)-periodic dynamics within a general time-scale framework rather than restricting attention to purely continuous settings. The notion of (ω, c)-periodicity encompasses several well-known types of periodic behavior as special cases, including classical periodicity, antiperiodicity, Bloch-type periodicity, as well as bounded and unbounded scaled periodicity. As such, it provides a unified and robust framework for understanding a wide variety of repetitive phenomena across different time scales. It is

worth noting that in practical applications, exact (ω, c)-periodicity may not always be observed, as real-world systems are often influenced by perturbations, uncertainties, or external disturbances. Nevertheless, the concept remains highly relevant, as it can be extended to describe approximate or generalized (ω, c)-periodic behavior, allowing for deviations within a controlled margin of error. This flexibility enables a more nuanced and realistic analysis of systems that exhibit nearly (ω, c) periodic dynamics. Motivated by these observations, this study focuses on (ω, c)-periodic functions and their generalizations on periodic time scales, with applications to the qualitative analysis of dynamic equations on time scales. Special attention is given to population models and neural networks, where such generalized periodic structures naturally arise and play a significant role in characterizing long-term system behavior. In realistic environments, however, system behavior is often affected by stochastic perturbations arising from environmental fluctuations, measurement noise, or intrinsic system randomness. Accordingly, this study further includes the analysis of (ω, c)-periodic stochastic processes, which capture recurrent dynamics in a statistical sense rather than in a strictly deterministic manner. The major contributions of the study can be summarized as follows.

Chapter 1 provides the necessary background, including an introduction to time-scale calculus, the motivation of the work, a review of related literature, and the mathematical preliminaries required for the subsequent chapters.

Bridging the gap between continuous and discrete systems, **Chapter 2** presents a detailed characterization of (ω, c)-periodic functions on periodic time scales and examines their fundamental properties. Several classes of delayed population models are analyzed to establish the existence of exponentially stable (ω, c)-periodic solutions, including Nicholson's model, Lasota–Ważewska model, and models incorporating a combination of both. The theoretical results are further illustrated and validated through numerical examples.

Chapter 3 investigates nonautonomous abstract dynamic equations on ω-periodic time scales, focusing on the existence of stable (ω, c)-periodic mild solutions. Sufficient conditions guaranteeing the existence, uniqueness and exponential stability of (ω, c)-periodic mild solutions for linear and semilinear abstract dynamic equations are derived using fixed point theory, Gronwall's inequality, semigroup theory, dichotomy theory, properties of (ω, c)-periodicity, and calculus on time scales. This study also delves into the continuous dependence of these exponentially stable (ω, c)-periodic mild solutions on the source term within a class of nonautonomous abstract semilinear dynamic equations.

Chapter 4 introduces the concept of (ω, c)-asymptotic periodicity on time scales, characterizing functions that do not exhibit (ω, c)-periodicity at early stages but eventually evolve into (ω, c)-periodic behavior.

Recognizing the importance of qualitative analysis in time-delayed systems, this work investigates a class of delayed cellular neural networks (DCNNs). Under appropriate assumptions on the coefficients and activation functions, sufficient conditions for the existence of stable (ω, c)-asymptotically periodic solutions for DCNNs are derived. The analysis relies fundamentally on the Banach fixed point theorem, combined with time-scale inequalities and composition results for (ω, c)-asymptotically periodic functions on time scales. Numerical examples and simulations are used to validate the proposed results across various time scales.

Further generalizing the notion of (ω, c)-asymptotic periodicity, **Chapter 5** provides the concept weighted pseudo-(ω, c)-periodicity on time scales. This chapter deals with a class of delayed neutral-type evolution systems with piecewise impulsive effects on time scales, which can be used to model a general and more realistic physical phenomenon in several situations. This system is investigated for the existence of (doubly) weighted pseudo-(ω, c)-periodic solutions. Using Lyapunov function techniques, the global exponential synchronization of weighted pseudo-(ω, c)-periodic solutions is further obtained for this system. Numerical examples are used to demonstrate that the obtained results effectively unify continuous and discrete analysis.

Chapter 6 develops a theory for p^{th}-mean S-asymptotically (ω, c)-periodic stochastic processes on periodic time scales and applies it to stochastic shunting-inhibitory cellular neural networks characterized by discrete time-varying delays and infinite distributed delays. In the presence of standard Lipschitz and growth conditions, as well as stochastic perturbations driven by a Wiener process, the existence and uniqueness of stable p^{th}-mean S-asymptotically (ω, c)-periodic solutions are established. The analysis utilizes time scale calculus, a Banach space setup that is appropriate for the (ω, c)-weighted processes, fixed-point arguments, and estimates of the Burkholder–Davis–Gundy type inequality for the stochastic integrals on time scales. The results of the theoretical analysis are illustrated and validated through the use of numerical simulations on representative continuous, discontinuous, and nonuniform time scales.

Chapter 7 presents the conclusions of the study, outlines directions for future research, and is followed by the bibliography.

Keywords: Time Scales, (ω, c)-periodicity, asymptotic (ω, c)-periodicity, Weighted pseudo (ω, c)-periodicity, S-asymptotic (ω, c)-periodicity, Stochastic Process, Evolution System, Cellular Neural networks, Nicholson's Blowflies model, Lasota–Ważewska model, Abstract dynamic equation, Impulses, Existence and uniqueness, Exponential Stability, Synchronization, Fixed point theory, Lyapunov function, Semigroup theory, Dichotomy theory.