Numerical Analysis and Simulation of Complex Financial Derivatives Pricing Models Governed by Partial Differential Equations

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Abstract:

Black and Scholes formulated a diffusion model for option pricing, which is based on Brownian motion and normal distribution. In the Black-Scholes model, the assumption of a constant volatility parameter does not adequately capture the volatility skews and smiles observed in the financial markets. Several jump-diffusion models have been developed as alternatives to the Black-Scholes model. Merton's and Kou's jump-diffusion models are used in financial markets to accurately capture the empirical phenomena of volatility smile and heavy tails, which are observed in real-world financial markets. In Merton's model, the log return of the underlying asset is driven by jump processes with normally distributed jump sizes, while Kou's model assumes that the jump sizes follow a double exponential distribution. These models are finite jump activity models, where the number of jumps in the stochastic process of the underlying asset is finite. In contrast, infinite activity models, such as the Carr-Geman-Madan-Yor model and the hyperbolic model, allow for an infinite number of jumps in the stochastic process of the underlying asset and are based on the Lévy process.

Next, we have considered the regime-switching jump-diffusion (RSJD) model for the valuation of financial derivatives. These models assume that the model parameters depend on a Markov chain and can have different values in different regimes. In the absence of regime-switching, option pricing under the jump-diffusion model involves solving a single partial integro-differential equation (PIDE). However, option pricing under regime-switching processes without jumps involves solving a system of partial differential equations (PDEs). Also, introducing non-local jump terms in the regime-switching model is equivalent to solving a system of PIDEs. Furthermore, we have also considered an irreversible investment decision problem on a finite time horizon where an instantaneous cash flow process of a firm follows a RSJD model. The value of a project can be derived from a PIDE and then we obtain a closed-form solution of the PIDE. It is proved that the value of the project converges to the solution on an infinite time horizon problem as the lifetime of the project tends to infinity. These details are introduced in Chapter 1.

Chapter 2 introduces novel implicit-explicit Runge-Kutta type methods for numerically solving PIDEs in option pricing under jump-diffusion models. These methods circumvent the need for numerical or analytical inversion of the coefficient matrix. Pricing European options involves solving a PIDE, while American options result in a linear complementarity problem (LCP), tackled using an operator splitting technique. The stability and convergence of these methods are established using the discrete l2-norm. To validate the efficiency and accuracy, the methods are applied to pricing European and American options

under Merton’s and Kou’s jump-diffusion models, and the computed results are compared with existing literature.

In Chapter 3, we develop second-order accurate implicit-explicit methods for PIDEs with non-smooth payoff functions. These methods implicitly handle differential and non-local integral (jump) terms in PIDEs. European options are priced by solving these PIDEs, while American options utilize LCPs involving the same operator. An operator splitting method is applied to address the LCP efficiently. Discontinuities in payoff functions can introduce numerical inaccuracies, impacting hedging parameter estimates. To mitigate this, a smoothing technique is combined with the proposed method, achieving high accuracy. Stability and convergence analysis of the method are also established using l2-norm. Finally, we present the numerical results using both uniform and non-uniform meshes to validate the method's accuracy.

Chapter 4 extends these numerical techniques to valuing financial derivatives under state-dependent RSJD models. An implicit technique is employed to solve these problems without inverting the coefficient matrix for each economic state. The pricing of European options under the RSJD process involves coupled PIDEs, while American options rely on coupled LCPs. The implicit-explicit methods, combined with operator splitting, address the LCPs. Theoretical consistency and convergence of these methods are shown using the discrete l2-norm. The accuracy and efficiency are validated through pricing European and American options across different economic states under RSJD models, with comparisons to recent literature methods.

In Chapter 5, we analyze and simulate an irreversible investment decision problem on a finite time horizon, where the cash flow process follows an RSJD model. It is shown both theoretically and numerically that project value converges to the infinite time horizon solution as the project lifetime increases. The real option value function with a finite expiration date is evaluated by solving an LCP under the RSJD model. Numerical experiments using the implicit-explicit Runge-Kutta method combined with operator splitting confirm a second-order convergence rate in the discrete l2-norm for temporal and spatial variables. The method's stability is also established, and experiments are conducted to determine optimal investment time. Chapter 6 concludes with a summary of findings and future research directions.

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