Extended Abstract

NUMERICAL ANALYSIS OF RADIAL BASIS FUNCTION BASED IMPLICIT-EXPLICIT FINITE DIFFERENCE METHODS FOR OPTION PRICING PROBLEMS

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Abstract

Radial basis functions (RBFs) are highly effective for approximating multivariable functions through linear combinations of univariate radial functions that depend only on the distance from a center. Their mesh-free nature makes them naturally suitable for solving problems. They are particularly useful when the dataset is known only at a finite set of discrete points, providing a smooth approximation that can be evaluated quickly and repeatedly. Beyond their role in function approximations, RBFs have also emerged as a robust tool for advancing higher-order numerical methods for solving complex partial differential equations (PDEs), partial integro-differential equations (PIDEs), and linear complementarity problems (LCPs) particularly in computational finance. Their global support and inherent smoothness make RBFs an appropriate choice for achieving higher-order accuracy. By selecting suitable RBFs from the various types available in literarture and by optimizing the shape parameters, these methods can attain high convergence rates while significantly reducing numerical errors.

In this thesis, we developed a radial basis function based finite difference (RBF-FD) scheme for solving various types of option pricing problems. Spatial discretization using RBF-FD method is combined with implicit-explicit (IMEX) temporal schemes to provide a reliable and efficient framework for computational finance problems with improved accuracy. Stability analyses for the proposed time semi-discretization methods are also presented. In addition, numerical experiments are performed to demonstrate the order of convergence, accuracy, and efficiency of the proposed numerical schemes. These RBF-FD–IMEX methods are efficiently applied to several stochastic models arising in option pricing. Specifically, we consider cases where the underlying asset follows a jump-diffusion process with local volatility, European and American options in an extended Markovian regime-switching jump-diffusion (RSJD) framework, Asian options governed by moving-boundary partial integro-differential equations with regime-switching jump-diffusion, and European and American options under liquidity shocks modeled through PDE system and semi-linear complementarity problems. These contributions establish high order, computationally efficient RBF-FD methods, capable of handling jumps, regime switching, path dependence, and liquidity effects, with direct application in option pricing.

In **Chapter 1**, a brief overview of financial markets and derivative securities is provided, with a particular focus on options and the fundamental concepts underlying them. The chapter also includes a literature review of recent numerical methods for option pricing and an introduction to the basic principles of radial basis function approximation. It presents the motivation of the study and outlines the problems addressed,

together with the preliminaries that will be used extensively in the subsequent chapters.

Chapter 2 presents efficient and accurate RBF-FD–IMEX methods for pricing options when the underlying asset follows a jump-diffusion process with local volatility. For time semi-discretization, we introduce three numerical techniques: Crank–Nikolson Leap-Frog (CNLF), Crank-Nikolson Adam-Bashforth (CNAB), and the second order Backward difference formula (BDF2), all incorporated with the RBF-FD method. The stability of the time semi-discretized schemes is also analyzed. The computational methods developed for European options are extended to American options. In particular, the RBF-FD IMEX methods are integrated with an operator splitting (OS) technique to solve the linear complementarity problem (LCP) with variable parameters, which determines the price of an American option. To validate the effectiveness and accuracy of the proposed techniques, numerical results for European and American put options under the Merton and Kou models are presented. Part of the work in this chapter has been published in *Engineering Analysis with Boundary Elements*.

Chapter 3 introduces three RBF-FD IMEX methods for pricing European and American options in an extended Markovian regime-switching jump-diffusion (RSJD) economy. A PIDE is used to compute European option values, while a linear complementarity problem (LCP) determines American option prices. To solve the LCP, the proposed scheme is combined with operator splitting methods, eliminating the need for fixed-point iterations at each economic stage and time step. The stability of the proposed time discretization methods is analyzed, and numerical experiments are used to validate the second-order convergence and computational efficiency of the three IMEX methods (BDF2, CNAB, CNLF) under the extended RSJD model. Part of the work in this chapter has been published in *Numerical Algorithms*.

Chapter 4 investigates the construction of a RBF-FD IMEX method for solving a moving-boundary PIDE system governing regime-switching jump-diffusion for Asian option pricing. The RBF-FD scheme is employed for spatial discretization and paired with the IMEX schemes for temporal discretization. The stability of the time semi-discretization method is also established theoretically. Numerical examples are presented to demonstrate the efficacy of the proposed scheme in terms of convergence and accuracy. Part of the work in this chapter has been published in *Engineering Analysis with Boundary Elements*.

Chapter 5 introduces two accurate and efficient finite difference RBF–FD methods for pricing European and American options under liquidity shocks. The problems are formulated as semi-linear complementarity and PDE systems for American and European options, respectively. For temporal semi-discretization, two backward difference formulas of order one and two (BDF1 & BDF2) are employed. The stability and convergence properties of the proposed methods are analyzed in detail. For American options, the semi-linear system of complementarity problems is solved by combining the RBF-FD approach with an operator splitting (OS) method. Numerical examples are provided for both European and American call options, and the results are validated against existing literature. Additionally, Greeks (Delta and Gamma) plots are presented for the American options to illustrate the sensitivity of option prices. Part of the work in this chapter has been published in *International Journal of Computer Mathematics*.

Chapter 6 is the concluding chapter, in which the contributions of the thesis are summarized and directions for future research are outlined.

Keywords

Option pricing; European Option; American Option; Asian Option; Radial basis function; Implicit-explicit finite difference methods; Stability analysis.

List of Publications

Thesis Publications

- **Rajesh Yadav**, Deepak Kumar Yadav, Alpesh Kumar: RBF based some implicit–explicit finite difference schemes for pricing option under extended jump-diffusion model. *Engineering Analysis with Boundary Elements* 156, 392-406 (2023). https://doi.org/10.1016/j.enganabound.2023.08.021

- **Rajesh Yadav**, Deepak Kumar Yadav, Alpesh Kumar: RBF-FD based some implicit-explicit methods for pricing option under regime-switching jump-diffusion model with variable coefficients. *Numerical Algorithms* 97(2), 645-685 (2024). https://doi.org/10.1007/s11075-023-01719-2

- Alpesh Kumar, Gobinda Rakshit, Deepak Kumar Yadav, **Rajesh Yadav**: RBF based some IMEX finite difference schemes for pricing option under liquidity switching. *International Journal of Computer Mathematics* 101(7), 768-788 (2024). https://doi.org/10.1080/00207160.2024.2383757

- Alpesh Kumar, Gobinda Rakshit, Deepak Kumar Yadav, **Rajesh Yadav**: Radial basis function-based finite difference schemes for pricing Asian options under a regime-switching jump diffusion model. *Engineering Analysis with Boundary Elements* 179, 106400 (2025). https://doi.org/10.1016/j.enganabound.2025.106400

Other Publications

- Alpesh Kumar, Gobinda Rakshit, Deepak Kumar Yadav, **Rajesh Yadav**: Computation and analysis of an implicit-explicit backward difference operator splitting method for pricing American options under the liquidity shocks. (*Under Review*)

- **Rajesh Yadav**, Deepak Kumar Yadav, Alpesh Kumar: Adaptive multiquadratic radial basis function-based explicit Runge-Kutta methods. (*Under Review*)